



TEXTBOOK SERIES  
FROM EUROPA-LEHRMITTEL  
for the metalworking trades

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# Mechanical and Metal Trades Handbook

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All batches of this edition may be used concurrently in the classroom since they are unchanged, except for some corrections to typographical errors and slight changes in standards.

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## Preface

The Mechanical and Metal Trades Handbook is well-suited for shop reference, tooling, machine building, maintenance and as a general book of knowledge. It is also useful for educational purposes, especially in practical work or curricula and continuing education programs.

### Target Groups

- Industrial and trade mechanics
- Technical product designers
- Apprentices in above trade areas
- Practitioners in trades and industry
- Mechanical engineering students

### Notes for the user

The contents of this book include tables and formulas in seven chapters as well as a table of contents, a subject index, a standard index and an international material comparison chart.

For a better overview, each of the seven chapters is preceded by an additional one-page **table of contents**.

The **tables** contain the most important guidelines, designs, types, dimensions and standard values for their respective subject areas.

Units are not specified in the legends for the **formulas** if several units are possible. However, the calculation examples for each formula use those units normally applied in practice.

### Changes in the 4<sup>th</sup> edition

The standards in this edition are **current as of January 2017**. Due to new standards and technical developments, the following contents have been updated, expanded or newly added:

- Quality management and environmental management according to the latest standard. Elimination of general terms from quality management.
- Introduction to "Geometrical product specification (GPS)" for technical communication principles.
- Additional tools and partially updated standard values for machining operations.
- Additions in cost accounting.
- Representation of structuring principles and reference designation in schematic circuit diagrams according to ISO 1219 and DIN EN 81346.

**Authors and publishers continue to be grateful to all users of the table handbook for notes and suggestions for improvement addressed to [lektorat@europa-lehrmittel.de](mailto:lektorat@europa-lehrmittel.de).**

Summer 2018

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## Standards and other regulations

### Standardization and standards terms

Standardization is the systematic achievement of uniformity of material and non-material objects, such as components, calculation methods, process flows and services for the benefit of the general public.


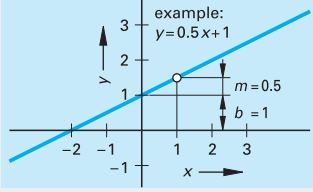
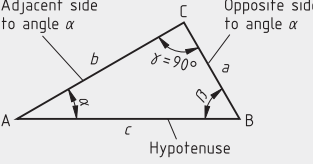
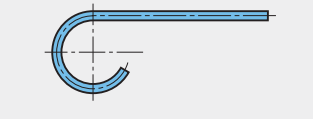

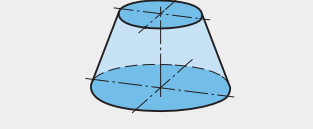
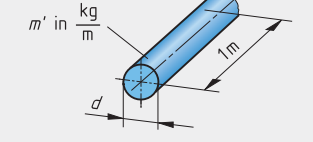
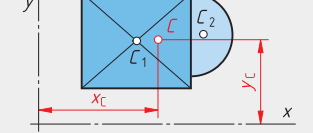
Standards term	Example	Explanation
Standard	DIN 509	A standard is the published result of the standardization work. Example: DIN 509 with shapes and dimensions of undercuts for turned parts and bores.
Part	DIN 30910-2	Standards can comprise several parts associated with each other. The part numbers are appended to the main standard number with hyphens. DIN 30910-2 describes sintered materials for filters for example, whereas Part 3 and 4 deal with sintered materials for bearings and formed parts.
Supplement	DIN 743 Suppl. 1	A supplement contains information for a standard, however no additional specifications. The supplement DIN 743 Suppl. 1, for example, contains application examples of load capacity calculations for shafts and axles described in DIN 743.
Draft	E DIN EN 10027-2 (2013-09)	Draft standards are made available to the public for examination and commenting. The new version of DIN EN 10027-2 (2015-07) with material codes for steels was available to the public as a draft for objections, for example, from September 2013 to February 2014.
Preliminary standard	DIN V 45696-1 (2006-02)	A preliminary standard contains the results of standardization, which have not been released as a standard because of certain provisos. For example, DIN V 45696-1 contains technical measures for the design of machines that transmit whole-body vibrations to humans.
Output date	DIN 76-1 (2004-06)	Date of publication which is made public in the DIN publication guide; this is the date at which time the standard becomes valid. DIN 76-1, which sets undercuts for metric ISO threads has been valid since June 2004 for example.

### Types of standards and regulations (selection)

Type	Abbreviation	Explanation	Purpose and contents
International standards (ISO standards)	ISO	International Organization for Standardization, Geneva (O and S are reversed in the abbreviation)	Simplifies the international exchange of goods and services as well as cooperation in scientific, technical and economic areas.
European standards (EN standards)	EN	European Committee for Standardization (Comité Européen de Normalisation), Brussels	Technical harmonization and the associated reduction of trade barriers for the advancement of the European market and the coalescence of Europe.
German standards (DIN standards)	DIN	Deutsches Institut für Normung e.V., Berlin (German Institute for Standardization)	National standardization facilitates rationalization, quality assurance, environmental protection and common understanding in economics, technology, science, management and public relations.
	DIN EN	European standard for which the German version has attained the status of a German standard	
	DIN ISO	German standard for which an international standard has been adopted without change.	
	DIN EN ISO	European standard for which an international standard has been adopted unchanged and the German version has the status of a German standard.	
	DIN VDE	Printed publication of the VDE, which has the status of a German standard.	
VDI guidelines	VDI	Verein Deutscher Ingenieure e.V., Düsseldorf (Association of German Engineers)	These guidelines give an account of the current state of the art in specific subject areas and contain, for example, concrete procedural guidelines for the performing calculations or designing processes in mechanical or electrical engineering.
VDE printed-publications	VDE	Verband der Elektrotechnik (German Association of Electrical Engineering) Elektronik Informationstechnik e.V., Frankfurt am Main (Association for Electronics Information Technology)	Recommendations in the area of quality technology.
DGQ publications	DGQ	Deutsche Gesellschaft für Qualität e.V., Frankfurt (German Society for Quality)	Recommendations in the area of production and work planning.
REFA sheets	REFA	Verband für Arbeitsstudien (Association for Work Design) REFA e.V., Darmstadt (Industrial Organization and Corporate Development)	



## 1 Technical Mathematics

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## Units of measurement

TM

SI<sup>1)</sup> base quantities and base units

cf. DIN 1301-1 (2010-10), -2 (1978-02), -3 (1979-10)

Base quantity	Length	Mass	Time	Electric current	Thermo-dynamic temperature	Amount of substance	Luminous intensity
Base units	metre	kilogramme	second	ampere	kelvin	mole	candela
Unit symbol	m	kg	s	A	K	mol	cd

<sup>1)</sup> The units for measurement are defined in the International System of Units SI (Système International d'Unités). It is based on the seven basic units (SI units), from which other units are derived.

## Base quantities, derived quantities and their units

Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
<b>Length, Area, Volume, Angle</b>					
Length	<i>l</i>	<b>metre</b>	m	1 m = 10 dm = 100 cm = 1000 mm 1 mm = 1000 μm 1 km = 1000 m	1 inch = 25.4 mm In aviation and nautical applications the following applies: 1 international nautical mile = 1852 m
Area	<i>A, S</i>	square metre are hectare	m <sup>2</sup> a ha	1 m <sup>2</sup> = 10 000 cm <sup>2</sup> = 1 000 000 mm <sup>2</sup> 1 a = 100 m <sup>2</sup> 1 ha = 100 a = 10 000 m <sup>2</sup> 100 ha = 1 km <sup>2</sup>	Symbol <i>S</i> only for cross-sectional areas Ar and hectare only for land
Volume	<i>V</i>	cubic metre litre	m <sup>3</sup> l, L	1 m <sup>3</sup> = 1000 dm <sup>3</sup> = 1 000 000 cm <sup>3</sup> 1 l = 1 L = 1 dm <sup>3</sup> = 10 dl = 0.001 m <sup>3</sup> 1 ml = 1 cm <sup>3</sup>	Mostly for fluids and gases
Plane angle (angle)	$\alpha, \beta, \gamma \dots$	radian degrees minutes seconds	rad ° ' "	1 rad = 1 m/m = 57.2957...° = 180°/π 1° = $\frac{\pi}{180}$ rad = 60' 1' = 1°/60 = 60" 1" = 1'/60 = 1°/3600	1 rad is the angle formed by the intersection of a circle around the centre of 1 m radius with an arc of 1 m length. In technical calculations instead of $\alpha = 33^\circ 17' 27.6''$ it is better to use $\alpha = 33.291^\circ$ .
Solid angle	$\Omega$	steradian	sr	1 sr = 1 m <sup>2</sup> /m <sup>2</sup>	The solid angle of 1 sr encompasses a sphere of $r = 1$ m on the surface, which corresponds to the area of a spherical segment of $A_0 = 1$ m <sup>2</sup> .
<b>Mechanics</b>					
Mass	<i>m</i>	<b>kilogramme</b> gram megagram metric ton	kg g Mg t	1 kg = 1000 g 1 g = 1000 mg 1 t = 1000 kg = 1 Mg 0.2 g = 1 ct	In everyday language, the mass of a solid is also referred to as weight. Mass for precious stones in carat (ct).
Linear mass density	<i>m'</i>	kilogramme per metre	kg/m	1 kg/m = 1 g/mm	For calculating the mass of bars, pro-files, pipes.
Area mass density	<i>m''</i>	kilogramme per square metre	kg/m <sup>2</sup>	1 kg/m <sup>2</sup> = 0.1 g/cm <sup>2</sup>	To calculate the mass of sheet metal.
Density	$\rho$	kilogramme per cubic metre	kg/m <sup>3</sup>	1000 kg/m <sup>3</sup> = 1 metric t/m <sup>3</sup> = 1 kg/dm <sup>3</sup> = 1 g/cm <sup>3</sup> = 1 g/ml = 1 mg/mm <sup>3</sup>	Density = Mass of a substance per volume unit For homogenous solids, the density is a location-independent quantity.

## Units of measurement

TM

Quantities and units (continued)					
Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
<b>Mechanics</b>					
Moment of inertia, 2nd moment of mass	$J$	kilogramme x square metre	$\text{kg} \cdot \text{m}^2$	The following applies to homogenous full cylinders with a mass $m$ and radius $r$ : $J = \frac{1}{2} \cdot m \cdot r^2$	The moment of inertia indicates the resistance of a rigid homogenous solid against the change in its rotational movement along the axis of rotation.
Force	$F$	newton	N	$1 \text{ N} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 1 \frac{\text{J}}{\text{m}}$	The force 1 N effects a change in velocity of 1 m/s in 1 s in a 1 kg mass.
Weight	$F_W, W$			$1 \text{ MN} = 10^3 \text{ kN} = 1\,000\,000 \text{ N}$	
Torque Bend. mom. Tors. mom.	$M$ $M_b$ $M_T, T$	newton x metre	$\text{N} \cdot \text{m}$	$1 \text{ N} \cdot \text{m} = 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$	$1 \text{ N} \cdot \text{m}$ is the moment that a force of 1 N effects with a lever arm of 1 m.
Momentum	$p$	kilogramme x metre per second	$\text{kg} \cdot \text{m/s}$	$1 \text{ kg} \cdot \text{m/s} = 1 \text{ N} \cdot \text{s}$	The momentum is the product of the mass times velocity. It has the direction of the velocity.
Pressure	$p$	pascal	Pa	$1 \text{ Pa} = 1 \text{ N/m}^2 = 0.01 \text{ mbar}$ $1 \text{ bar} = 100\,000 \text{ N/m}^2 = 10 \text{ N/cm}^2 = 10^5 \text{ Pa}$	Pressure refers to the force per unit area. For gage pressure, the symbol $p_g$ is used (DIN 1314). $1 \text{ bar} = 14.5 \text{ psi}$ (pounds per square inch)
Mechanical stress	$\sigma, \tau$	newton per square millimetre	$\text{N/mm}^2$	$1 \text{ mbar} = 1 \text{ hPa}$ $1 \text{ N/mm}^2 = 10 \text{ bar} = 1 \text{ MN/m}^2 = 1 \text{ MPa}$ $1 \text{ daN/cm}^2 = 0.1 \text{ N/mm}^2$	$1 \text{ bar} = 14.5 \text{ psi}$ (pounds per square inch)
Second moment of area	$I$	metre to the fourth power centimetre to the fourth power	$\text{m}^4$ $\text{cm}^4$	$1 \text{ m}^4 = 100\,000\,000 \text{ cm}^4$	Previously: Geometrical moment of inertia
Energy, Work, Quantity of heat	$E, W$	joule	J	$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ W} \cdot \text{s} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$	Joule for all forms of energy, kW·h preferred for electrical energy.
Power, Heat flow	$P$ $\Phi$	watt	W	$1 \text{ W} = 1 \text{ J/s} = 1 \text{ N} \cdot \text{m/s} = 1 \text{ V} \cdot \text{A} = 1 \text{ m}^2 \cdot \text{kg/s}^3$	Power describes the work which is achieved within a specific time.
<b>Time</b>					
Time, Time span, Duration	$t$	<b>seconds</b> minutes hours day year	s min h d a	$1 \text{ min} = 60 \text{ s}$ $1 \text{ h} = 60 \text{ min} = 3600 \text{ s}$ $1 \text{ d} = 24 \text{ h} = 86\,400 \text{ s}$	3 h means a time span (3 hrs.), $3^{\text{h}}$ means a point in time (3 o'clock). If points in time are written in mixed form, e.g. $3^{\text{h}}24^{\text{m}}10^{\text{s}}$ , the symbol min can be shortened to m.
Frequency	$f, \nu$	hertz	Hz	$1 \text{ Hz} = 1/\text{s}$	$1 \text{ Hz} \hat{=} 1 \text{ cycle in 1 second.}$
Rotational speed, Rotational frequency	$n$	1 per second 1 per minute	1/s 1/min	$1/\text{s} = 60/\text{min} = 60 \text{ min}^{-1}$ $1/\text{min} = 1 \text{ min}^{-1} = \frac{1}{60 \text{ s}}$	The number of revolutions per unit of time gives the revolution frequency, also called rpm.
Velocity	$v$	metre per second metre per minute kilometre per hour	m/s m/min km/h	$1 \text{ m/s} = 60 \text{ m/min} = 3.6 \text{ km/h}$ $1 \text{ m/min} = \frac{1 \text{ m}}{60 \text{ s}}$ $1 \text{ km/h} = \frac{1 \text{ m}}{3.6 \text{ s}}$	Speed for nautical journeys in knots (kn): $1 \text{ kn} = 1.852 \text{ km/h}$ mile per hour = 1 mile/h = 1 mph $1 \text{ mph} = 1.60934 \text{ km/h}$
Angular velocity	$\omega$	1 per second radians per second	1/s rad/s	$\omega = 2 \pi \cdot n$	For a rpm of $n = 2/\text{s}$ the angular velocity $\omega = 4 \pi/\text{s}$ .
Acceleration	$a, g$	metre per second squared	$\text{m/s}^2$	$1 \text{ m/s}^2 = \frac{1 \text{ m/s}}{1 \text{ s}}$	Symbol $g$ only for acceleration due to gravity. $g = 9.81 \text{ m/s}^2 \approx 10 \text{ m/s}^2$

## Units of measurement

TM

Quantities and units (continued)					
Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
<b>Electricity and magnetism</b>					
Electric current	$I$	<b>ampere</b>	A		A moving electrical charge is called electricity. The electromotive force is equal to the potential difference between two points in an electric field. The reciprocal of the electrical resistance is called the electrical conductivity.
Electromotive force	$U$	volt	V	$1 \text{ V} = 1 \text{ W}/1 \text{ A} = 1 \text{ J}/\text{C}$	
Elect. resistance	$R$	ohm	$\Omega$	$1 \Omega = 1 \text{ V}/1 \text{ A}$	
Elect. conductance	$G$	siemens	S	$1 \text{ S} = 1 \text{ A}/1 \text{ V} = 1/\Omega$	
Specific resistance	$\rho$	ohm x metre	$\Omega \cdot \text{m}$	$10^{-6} \Omega \cdot \text{m} = 1 \Omega \cdot \text{mm}^2/\text{m}$	$\rho = \frac{1}{\kappa} \text{ in } \frac{\Omega \cdot \text{mm}^2}{\text{m}}$ $\kappa = \frac{1}{\rho} \text{ in } \frac{\text{m}}{\Omega \cdot \text{mm}^2}$
Conductivity	$\gamma, \kappa$	siemens per metre	S/m		
Frequency	$f$	hertz	Hz	$1 \text{ Hz} = 1/\text{s}$ $1000 \text{ Hz} = 1 \text{ kHz}$	Frequency of public electric utility: EU 50 Hz, USA/Canada 60 Hz
Elect. Work	$W$	joule	J	$1 \text{ J} = 1 \text{ W} \cdot \text{s} = 1 \text{ N} \cdot \text{m}$ $1 \text{ kW} \cdot \text{h} = 3,6 \text{ MJ}$ $1 \text{ W} \cdot \text{h} = 3,6 \text{ kJ}$	In atomic and nuclear physics the unit eV (electron volt) is used.
Phase difference	$\varphi$	–	–	for alternating current: $\cos \varphi = \frac{P}{U \cdot I}$	The angle between current and voltage in an inductive or capacitive load.
Elect. field strength	$E$	volts per metre	V/m		$E = \frac{F}{Q}, C = \frac{Q}{U}, Q = I \cdot t$
Elect. charge	$Q$	coulomb	C	$1 \text{ C} = 1 \text{ A} \cdot 1 \text{ s}; 1 \text{ A} \cdot \text{h} = 3,6 \text{ kC}$	
Elect. capacity	$C$	farad	F	$1 \text{ F} = 1 \text{ C}/\text{V}$	
Inductance	$L$	henry	H	$1 \text{ H} = 1 \text{ V} \cdot \text{s}/\text{A}$	
Power Effective power	$P$	watt	W	$1 \text{ W} = 1 \text{ J}/\text{s} = 1 \text{ N} \cdot \text{m}/\text{s}$ $= 1 \text{ V} \cdot \text{A}$	In electrical power engineering: Apparent power $S$ in V · A
<b>Thermodynamics and heat transfer</b>					
Quantity	Symbol	Unit Name	Symbol	Relationship	Remarks Examples of application
Thermodynamic temperature	$T, \theta$	<b>kelvin</b>	K	$0 \text{ K} = -273,15 \text{ }^\circ\text{C}$	Kelvin (K) and degrees Celsius ( $^\circ\text{C}$ ) are used for temperatures and temperature differences. $t = T - T_0; T_0 = 273,15 \text{ K}$ Conversion in $^\circ\text{F}$ : page 51
Celsius temperature	$t, \vartheta$	degrees Celsius	$^\circ\text{C}$	$0 \text{ }^\circ\text{C} = 273,15 \text{ K}$ $0 \text{ }^\circ\text{C} = 32 \text{ }^\circ\text{F}$ $0 \text{ }^\circ\text{F} = -17,77 \text{ }^\circ\text{C}$	
Quantity of heat	$Q$	joule	J	$1 \text{ J} = 1 \text{ W} \cdot \text{s} = 1 \text{ N} \cdot \text{m}$ $1 \text{ kW} \cdot \text{h} = 3600000 \text{ J} = 3,6 \text{ MJ}$	
Net calorific value	$H_u$	joule per kilogramme joule per cubic metre	J/kg J/m <sup>3</sup>	$1 \text{ MJ}/\text{kg} = 1000000 \text{ J}/\text{kg}$ $1 \text{ MJ}/\text{m}^3 = 1000000 \text{ J}/\text{m}^3$	Thermal energy released per kg fuel (or for each m <sup>3</sup> ) minus the heat of vaporisation of the water vapour contained in the exhaust gases.
<b>Non-SI units</b>					
Length	Area	Volume	Mass	Energy, power	
1 inch (in) = 25,4 mm	1 sq.in = 6,452 cm <sup>2</sup>	1 cu.in = 16,39 cm <sup>3</sup>	1 oz = 28,35 g	1 PSh = 0,735 kWh	
1 foot (ft) = 0,3048 m	1 sq.ft = 9,29 dm <sup>2</sup>	1 cu.ft = 28,32 dm <sup>3</sup>	1 lb = 453,6 g	1 PS = 0,7355 kW	
1 yard (yd) = 0,9144 m	1 sq.yd = 0,8361 m <sup>2</sup>	1 cu.yd = 764,6 dm <sup>3</sup>	1 t = 1000 kg	1 kcal = 4186,8 Ws	
1 nautical mile = 1,852 km	1 acre = 4046,856 m <sup>2</sup>	1 gallon (US) = 3,785 l	1 short ton = 907,2 kg	1 kcal = 1,166 Wh	
1 land mile = 1,6093 km	<b>Pressure, force</b>	1 gallon (UK) = 4,546 l	1 Karat = 0,2 g	1 kpm/s = 9,807 W	
	1 bar = 14,5 pound/in <sup>2</sup>	1 barrel = 158,8 l	1 pound/in <sup>3</sup> = 27,68 g/cm <sup>3</sup>	1 Btu = 1055 Ws	
	1 N/mm <sup>2</sup> = 145,038 pound/in <sup>2</sup>			1 hp = 745,7 W	

Formula symbols, mathematical symbols

TM

Formula symbols <span style="float: right;">cf. DIN 1304-1 (1994-03)</span>					
Formula symbol	Meaning	Formula symbol	Meaning	Formula symbol	Meaning
<b>Length, Area, Volume, Angle</b>					
$l$	Length	$r, R$	Radius	$\alpha, \beta, \gamma$	Planar angle
$w$	Width	$d, D$	Diameter	$\Omega$	Solid angle
$h$	Height	$A, S$	Area, Cross-sectional area	$\lambda$	Wave length
$s$	Linear distance	$V$	Volume		
<b>Mechanics</b>					
$m$	Mass	$F$	Force	$G$	Shear modulus
$m'$	Linear mass density	$F_{gr}, W$	Gravitational force, Weight	$\mu, f$	Coefficient of friction
$m''$	Area mass density	$T$	Torque	$W$	Section modulus
$\rho$	Density	$M_T, T$	Torsional moment	$I$	2nd moment of mass
$J$	Moment of inertia	$M_b$	Bending moment	$W, E$	Work, Energy
$p$	Pressure	$\sigma$	Normal stress	$W_{pot}, E_p$	Potential energy
$p_{abs}$	Absolute pressure	$\tau$	Shear stress	$W_k, E_k$	Kinetic energy
$p_{amb}$	Ambient pressure	$\epsilon$	Strain	$P$	Power
$p_g$	Gage pressure	$E$	Modulus of elasticity	$\eta$	Efficiency
<b>Time</b>					
$t$	Time, Duration	$f, \nu$	Frequency	$a$	Acceleration
$T$	Cycle duration	$v, u$	Velocity	$g$	Local gravitational acceleration
$n$	Revolution frequency, Speed	$\omega$	Angular velocity	$\alpha$	Angular acceleration
				$Q, \dot{V}, q_v$	Volumetric flow rate
<b>Electricity</b>					
$Q$	Electric charge, Quantity of electricity	$L$	Inductance	$X$	Reactance
$E$	Electromotive force	$R$	Resistance	$Z$	Impedance
$C$	Capacitance	$\rho$	Specific resistance	$\varphi$	Phase difference
$I$	Electric current	$\gamma, \kappa$	Electrical conductivity	$N$	Number of turns
<b>Heat</b>					
$T, \theta$	Thermodynamic temperature	$Q$	Heat, Quantity of heat	$\Phi, \dot{Q}$	Heat flow
$\Delta T, \Delta t, \Delta \theta$	Temperature difference	$\lambda$	Thermal conductivity	$a$	Thermal diffusivity
$t, \vartheta$	Celsius temperature	$\alpha$	Heat transition coefficient	$c$	Specific heat capacity
$\alpha, \alpha'$	Coefficient of linear expansion	$k$	Heat transmission coefficient	$H_{net}$	Net calorific value
<b>Light, Electromagnetic radiation</b>					
$E_v$	Illuminance	$f$	Focal length	$I_e$	Luminous intensity
		$n$	Refractive index	$Q_{er}, W$	Radiant energy
<b>Acoustics</b>					
$p$	Acoustic pressure	$L_p$	Acoustic pressure level	$N$	Loudness
$c$	Acoustic velocity	$I$	Sound intensity	$L_N$	Loudness level
<b>Mathematical symbols <span style="float: right;">cf. DIN 1302 (1999-12)</span></b>					
Math. symbol	Spoken	Math. symbol	Spoken	Math. symbol	Spoken
$\approx$	approximately the same	$\sim$	proportional	$\log$	logarithm (general)
$\dots$	corresponds to and so forth	$a^x$	a high x, xth of power of a	$\lg$	common logarithm
$\infty$	infinity	$\sqrt{\quad}$	square root of	$\ln$	natural logarithm
		$\sqrt[n]{\quad}$	nth root of	$e$	Euler number (e = 2.718281...)
$=$	equal to	$ x $	amount of x	$\sin$	sine
$\neq$	not equal to	$\perp$	vertical to	$\cos$	cosine
$\stackrel{\text{def}}{=}$	is the same by definition	$\parallel$	is parallel to	$\tan$	tangent
$<$	less than	$\parallel \uparrow$	parallel in same direction	$\cot$	cotangent
$\leq$	less than or equal to	$\uparrow \downarrow$	parallel in opposite direction	$(, [, \{$	parentheses, brackets
$>$	greater than	$\sphericalangle$	angle	$) , ] , \}$	open and closed
$\geq$	greater than or equal to	$\triangle$	triangle	$\pi$	pi (circle constant = 3.14159 ...)
$+$	plus	$\equiv$	congruent to		
$-$	minus	$\Delta x$	delta x (difference between two values)	$\overline{AB}$	line segment AB
$\cdot$	times, multiplied by			$\widehat{AB}$	arc AB
$-, /, :, \div$	over, divided by, per, to	$\%$	percent, of a hundred	$a', a''$	a prime, a double prime
$\Sigma$	sigma (summation)	$\text{‰}$	per mil, of a thousand	$a_1, a_2$	a sub 1, a sub 2

## Formulas, equations, graphs

### Formulas

In most cases, the calculation of physical quantities is done with the help of formulas. They consist of:

- Formula symbols, e.g.  $v_c$  for cutting velocity,  $d$  for diameter,  $n$  for speed
- Operators (calculation rules), e.g.  $\cdot$  for multiplication,  $+$  for addition,  $-$  for subtraction and  $\frac{\quad}{\quad}$  (fraction line) for division
- Constants, e.g.  $\pi$  (pi) = 3.14159 ...
- Numbers, e.g. 10, 15 ...

The formula symbols (page 13) are wildcards for quantities. When solving mathematical problems, the known quantities with their units are filled in the formulas. Before or during the calculation process, the units are converted in a way that

- the calculation becomes feasible or
- the result comprises the required unit.

Most quantities and units are standardised (page 10).

The **result** is always a **numerical value** accompanied by a **unit**, e.g. 4.5 m, 15 s

#### Formula for cutting velocity

$$v_c = \pi \cdot d \cdot n$$

#### Example:

What is the cutting velocity  $v_c$  in m/min for  $d = 200$  mm and  $n = 630$ /min?

$$v_c = \pi \cdot d \cdot n = \pi \cdot 200 \text{ mm} \cdot 630 \frac{1}{\text{min}} = \pi \cdot 200 \text{ mm} \cdot \frac{1 \text{ m}}{1000 \text{ mm}} \cdot 630 \frac{1}{\text{min}} = 395.84 \frac{\text{m}}{\text{min}}$$

### Numerical value equations

Numerical value equations or numerical equations are formulas in which the typical conversions of units have already been integrated. The following should be noted when using equations:

The numerical values of the individual quantities may only be used in combination with the designated unit.

- The units are not carried along in the calculation.
- The unit of the quantity to be obtained is predetermined.

#### Example:

What is the torque  $T$  of an electrical motor with a driving power of  $P = 15$  kW and a speed of  $n = 750$ /min?

$$T = \frac{9550 \cdot P}{n} = \frac{9550 \cdot 15}{750} \text{ N} \cdot \text{m} = 191 \text{ N} \cdot \text{m}$$

#### Numerical value equation for torque

$$T = \frac{9550 \cdot P}{n}$$

Designated unit	
Designation	Unit
$T$	Torque N · m
$P$	Power kW
$n$	Speed 1/min

### Equations and graphs

In functional equations,  $y$  is the function of  $x$ , with  $x$  as an independent and  $y$  as a dependent variable. The number pairs  $(x, y)$  of a value table form a graph in the  $x$ - $y$  system of coordinates.

#### Assigned function

$$y = f(x)$$

#### Linear function

$$y = m \cdot x + b$$

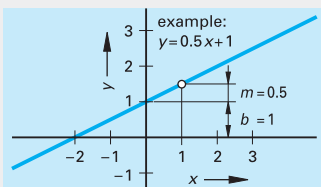
#### Examples:

##### Cost function

$$C_t = C_v \cdot Q + C_f$$

##### Revenue function

$$R = R/\text{piece} \cdot Q$$



#### 1st example:

$$y = 0.5x + 1$$

$x$	-2	0	2	3
$y$	0	1	2	2.5

#### 2nd example:

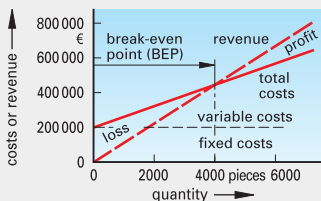
##### Cost function and revenue function

$$C_t = 60 \text{ €/piece} \cdot Q + 200\,000 \text{ €}$$

$$R = 110 \text{ €/piece} \cdot Q$$

$Q$	0	4000	6000
$C_t$	200000	440000	560000
$R$	0	440000	660000

- $C_t$  total costs → dependent variable
- $Q$  quantity → independent variable
- $C_f$  fixed costs →  $y$  coordinate section
- $C_v$  variable costs → gradient of the function
- $R$  revenue → dependent variable



## Transformation of formulas

### Transformation of formulas

Formulas and numerical equations are transformed so that the quantity to be obtained stands alone on the left side of the equation. The value of the left side and right side of the formula must not change during the transformation. The following rule applies to all steps of the formula transformation:

Changes applied to the left formula side	=	Changes applied to the right formula side
--	---	---

Formula

$$P = \frac{F \cdot s}{t}$$

left side of the formula	=	right side of the formula
--------------------------	---	---------------------------

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To be able to trace each step of the transformation, it is useful to mark it to the right next to the formula:

| ·  $t$  → both sides of the formula are multiplied by  $t$ .

| :  $F$  → both sides of the formula are divided by  $F$ .

### Transformations of sums

**Example:** Formula  $L = l_1 + l_2$ , transformation to find  $l_2$

$\boxed{1} \quad L = l_1 + l_2 \quad   -l_1 \quad \text{subtract } l_1$	$\boxed{3} \quad L - l_1 = l_2 \quad \text{invert both sides}$
$\boxed{2} \quad L - l_1 = l_1 + l_2 - l_1 \quad \text{perform subtraction}$	$\boxed{4} \quad l_2 = L - l_1 \quad \text{transformed formula}$

### Transformations of products

**Example:** Formula  $A = l \cdot b$ , transformation to find  $l$

$\boxed{1} \quad A = l \cdot b \quad   : b \quad \text{divide by } b$	$\boxed{3} \quad \frac{A}{b} = l \quad \text{invert both sides}$
$\boxed{2} \quad \frac{A}{b} = \frac{l \cdot b}{b} \quad \text{cancel } b$	$\boxed{4} \quad l = \frac{A}{b} \quad \text{transformed formula}$

### Transformations of fractions

**Example:** Formula  $n = \frac{l}{l_1 + s}$ , transformation to find  $s$

$\boxed{1} \quad n = \frac{l}{l_1 + s} \quad   \cdot (l_1 + s) \quad \text{multiply with } (l_1 + s)$	$\boxed{4} \quad n \cdot l_1 - n \cdot l_1 + n \cdot s = l - n \cdot l_1 \quad   : n \quad \text{subtract divide by } n$
$\boxed{2} \quad n \cdot (l_1 + s) = \frac{l \cdot (l_1 + s)}{(l_1 + s)} \quad \text{reduce right-side of formula solve the term in brackets}$	$\boxed{5} \quad \frac{s \cdot n}{n} = \frac{l - n \cdot l_1}{n} \quad \text{cancel } n$
$\boxed{3} \quad n \cdot l_1 + n \cdot s = l \quad   - n \cdot l_1 \quad \text{subtract } -n \cdot l_1$	$\boxed{6} \quad s = \frac{l - n \cdot l_1}{n} \quad \text{transformed formula}$

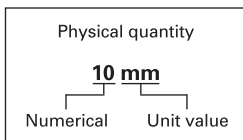
### Transformations of roots

**Example:** Formula  $c = \sqrt{a^2 + b^2}$ , transformation to find  $a$

$\boxed{1} \quad c = \sqrt{a^2 + b^2} \quad   ( )^2 \quad \text{square the formula}$	$\boxed{4} \quad a^2 = c^2 - b^2 \quad   \sqrt{\quad} \quad \text{square equation}$
$\boxed{2} \quad c^2 = a^2 + b^2 \quad   - b^2 \quad \text{subtract } b^2$	$\boxed{5} \quad \sqrt{a^2} = \sqrt{c^2 - b^2} \quad \text{simplify the expression}$
$\boxed{3} \quad c^2 - b^2 = a^2 + b^2 - b^2 \quad \text{subtract, invert both sides}$	$\boxed{6} \quad a = \sqrt{c^2 - b^2} \quad \text{transformed formula}$

## Quantities and units

### Numerical values and units



Physical quantities, e.g. 125 mm, consist of a

- **Numerical value**, which is determined by measurement or calculation, and a
- **Unit**, e.g. m, kg

Units are standardised in accordance with DIN 1301-1 (page 10).

Very large or very small numerical values can be represented in a simplified way as decimal multiples or factors with the help of prefixes, e.g. 0.004 mm = 4  $\mu$ m.

### Decimal multiples or factors of units

cf. DIN 1301-2 (1978-02)

Symbol	Prefix Name	Power of ten	Mathematical designation	Examples
T	tera	$10^{12}$	trillion	12 000 000 000 000 N = $12 \cdot 10^{12}$ N = 12 TN (teranewtons)
G	giga	$10^9$	billion	45 000 000 000 W = $45 \cdot 10^9$ W = 45 GW (gigawatts)
M	mega	$10^6$	million	8 500 000 V = $8.5 \cdot 10^6$ V = 8.5 MV (megavolts)
k	kilo	$10^3$	thousand	12 600 W = $12.6 \cdot 10^3$ W = 12.6 kW (kilowatts)
h	hecto	$10^2$	hundred	500 l = $5 \cdot 10^2$ l = 5 hl (hectolitres)
da	deca	$10^1$	ten	32 m = $3.2 \cdot 10^1$ m = 3.2 dam (decametres)
–	–	$10^0$	one	1.5 m = $1.5 \cdot 10^0$ m
d	deci	$10^{-1}$	tenth	0.5 l = $5 \cdot 10^{-1}$ l = 5 dl (decilitres)
c	centi	$10^{-2}$	hundredth	0.25 m = $25 \cdot 10^{-2}$ m = 25 cm (centimetres)
m	milli	$10^{-3}$	thousandth	0.375 A = $375 \cdot 10^{-3}$ A = 375 mA (milliamperes)
$\mu$	micro	$10^{-6}$	millionth	0.000 052 m = $52 \cdot 10^{-6}$ m = 52 $\mu$ m (micrometres)
n	nano	$10^{-9}$	billionth	0.000 000 075 m = $75 \cdot 10^{-9}$ m = 75 nm (nanometres)
p	pico	$10^{-12}$	trillionth	0.000 000 000 006 F = $6 \cdot 10^{-12}$ F = 6 pF (picofarads)

### Conversion of units

Calculations with physical units are only possible if these units refer to the same base in this calculation. When solving mathematical problems, units often must be converted to basic units, e.g. mm in m, h in s, mm<sup>2</sup> in m<sup>2</sup>. This is done with the help of conversion factors that represent the value 1 (coherent units).

### Conversion factors for units (excerpt)

Quantity	Conversion factors, e. g.	Quantity	Conversion factors, e. g.
Length	$1 = \frac{10 \text{ mm}}{1 \text{ cm}} = \frac{1000 \text{ mm}}{1 \text{ m}} = \frac{1 \text{ m}}{1000 \text{ mm}} = \frac{1 \text{ km}}{1000 \text{ m}}$	Time	$1 = \frac{60 \text{ min}}{1 \text{ h}} = \frac{3600 \text{ s}}{1 \text{ h}} = \frac{60 \text{ s}}{1 \text{ min}} = \frac{1 \text{ min}}{60 \text{ s}}$
Areas	$1 = \frac{100 \text{ mm}^2}{1 \text{ cm}^2} = \frac{100 \text{ cm}^2}{1 \text{ dm}^2} = \frac{1 \text{ cm}^2}{100 \text{ mm}^2} = \frac{1 \text{ dm}^2}{100 \text{ cm}^2}$	Angle	$1 = \frac{60'}{1^\circ} = \frac{60''}{1'} = \frac{3600''}{1^\circ} = \frac{1^\circ}{60 \text{ s}}$
Volume	$1 = \frac{1000 \text{ mm}^3}{1 \text{ cm}^3} = \frac{1000 \text{ cm}^3}{1 \text{ dm}^3} = \frac{1 \text{ cm}^3}{1000 \text{ mm}^3} = \frac{1 \text{ dm}^3}{1000 \text{ cm}^3}$	Inch	1 inch = 25.4 mm; 1 mm = $\frac{1}{25.4}$ inch

#### 1<sup>st</sup> example:

Convert volume  $V = 3416 \text{ mm}^3$  to  $\text{cm}^3$ .

Volume  $V$  is multiplied by a conversion factor. Its numerator has the unit  $\text{cm}^3$  and its denominator the unit  $\text{mm}^3$ .

$$V = 3416 \text{ mm}^3 = \frac{1 \text{ cm}^3 \cdot 3416 \text{ mm}^3}{1000 \text{ mm}^3} = \frac{3416 \text{ cm}^3}{1000} = 3.416 \text{ cm}^3$$

#### 2<sup>nd</sup> example:

The angle size specification  $\alpha = 42^\circ 16'$  is to be expressed in degrees ( $^\circ$ ).

The partial angle  $16'$  must be converted to degrees ( $^\circ$ ). The value is multiplied by a conversion factor, the numerator of which has the unit degree ( $^\circ$ ) and the denominator the unit minute ( $'$ ).

$$\alpha = 42^\circ + 16' \cdot \frac{1^\circ}{60'} = 42^\circ + \frac{16 \cdot 1^\circ}{60} = 42^\circ + 0.267^\circ = 42.267^\circ$$



## Calculation with quantities, percentage and interest calculation

TM

### Calculation with quantities

Physical quantities are mathematically treated as products.

#### • Adding and subtracting

Numerical values that have the same unit are added or subtracted and the unit is carried over to the result.

**Example:**

$$L = l_1 + l_2 - l_3 \text{ where } l_1 = 124 \text{ mm, } l_2 = 18 \text{ mm, } l_3 = 44 \text{ mm; } L = ?$$

$$L = 124 \text{ mm} + 18 \text{ mm} - 44 \text{ mm} = (124 + 18 - 44) \text{ mm} = \mathbf{98 \text{ mm}}$$

#### • Multiplying and dividing

The numerical values and the units correspond to the factors of products.

**Example:**

$$F_1 \cdot l_1 = F_2 \cdot l_2 \text{ where } F_1 = 180 \text{ N, } l_1 = 75 \text{ mm, } l_2 = 105 \text{ mm; } F_2 = ?$$

$$F_2 = \frac{F_1 \cdot l_1}{l_2} = \frac{180 \text{ N} \cdot 75 \text{ mm}}{105 \text{ mm}} = 128.57 \frac{\text{N} \cdot \text{mm}}{\text{mm}} = \mathbf{128.57 \text{ N}}$$

#### • Multiplying and dividing powers

Powers that have the same base are multiplied or divided by adding or subtracting their exponents.

**Example:**

$$W = \frac{A \cdot a^2}{e} \text{ with } A = 15 \text{ cm}^2, a = 7.5 \text{ cm, } e = 2.4 \text{ cm; } W = ?$$

$$W = \frac{15 \text{ cm}^2 \cdot (7.5 \text{ cm})^2}{2.4 \text{ cm}} = \frac{15 \cdot 56.25 \text{ cm}^{2+2}}{2.4 \text{ cm}^1} = 351.56 \text{ cm}^{4-1} = \mathbf{351.56 \text{ cm}^3}$$

#### Rules for raising to powers

$a$  base  
 $m, n \dots$  exponents

#### Multiplying powers

$$a^2 \cdot a^3 = a^{2+3}$$

$$a^m \cdot a^n = a^{m+n}$$

#### Dividing powers

$$\frac{a^2}{a^3} = a^{2-3}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

#### Special cases

$$a^{-2} = \frac{1}{a^2}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^1 = a$$

$$a^0 = 1$$

### Percentage calculation

The **percentage rate** indicates the part of the base value in hundredths. The **base value** is the value from which the percentage is to be calculated. The **percent value** is the amount representing the percentage of the base value.

$P_r$  percentage rate, in percent     $P_v$  percent value     $B_v$  base value

**Example:**

Weight of raw part: 250 kg (base value); material loss of 2% (percentage rate); material loss in kg = ? (percent value)

$$P_v = \frac{B_v \cdot P_r}{100\%} = \frac{250 \text{ kg} \cdot 2\%}{100\%} = \mathbf{5 \text{ kg}}$$

#### Percent value

$$P_v = \frac{B_v \cdot P_r}{100\%}$$

### Interest calculation

$P$  principle     $I$  interest     $t$  period in days, interest period  
 $A$  amount accumulated     $r$  interest rate per year

**1st example:**

$$P = 2800.00 \text{ €; } r = 6 \frac{\%}{a}; t = \frac{1}{2} a; I = ?$$

$$I = \frac{2800.00 \text{ €} \cdot 6 \frac{\%}{a} \cdot 0.5 a}{100\%} = \mathbf{84.00 \text{ €}}$$

**2nd example:**

$$P = 4800.00 \text{ €; } r = 5.1 \frac{\%}{a}; t = 50 \text{ d; } I = ?$$

$$I = \frac{4800.00 \text{ €} \cdot 5.1 \frac{\%}{a} \cdot 50 \text{ d}}{100\% \cdot 360 \frac{d}{a}} = \mathbf{34.00 \text{ €}}$$

#### Interest

$$I = \frac{P \cdot r \cdot t}{100\% \cdot 360}$$

1 interest year (1 a) = 360 days (360 d)

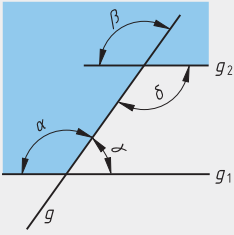
360 d = 12 months

1 interest month = 30 days

## Types of angles, theorem of intersecting lines, angles in a triangle, pythagorean theorem

TM

## Types of angles



- $g$  straight line  
 $g_1, g_2$  parallel straight lines  
 $\alpha, \beta$  corresponding angles  
 $\beta, \delta$  opposite angles  
 $\alpha, \delta$  alternate angles  
 $\alpha, \gamma$  adjacent angles

If two parallels are intersected by a straight line, there are geometrical interrelationships between the resulting angles.

## Corresponding angles

$$\alpha = \beta$$

## Opposite angles

$$\beta = \delta$$

## Alternate angles

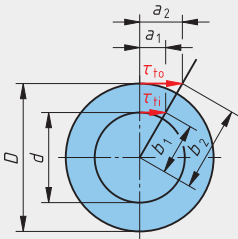
$$\alpha = \delta$$

## Adjacent angles

$$\alpha + \gamma = 180^\circ$$

## Theorem of intersecting lines

- $\tau_{to}$  outer torsional stress  
 $\tau_{ti}$  inner torsional stress



If two intersecting lines are intercepted by a pair of parallels, the resulting segments form equal ratios.

$$\begin{aligned} D &= 40 \text{ mm}, d = 30 \text{ mm}, \\ \tau_{to} &= 135 \text{ N/mm}^2; \tau_{ti} = ? \\ \frac{\tau_{ti}}{\tau_{to}} &= \frac{d}{D} \Rightarrow \tau_{ti} = \frac{\tau_{to} \cdot d}{D} \\ &= \frac{135 \text{ N/mm}^2 \cdot 30 \text{ mm}}{40 \text{ mm}} = 101.25 \text{ N/mm}^2 \end{aligned}$$

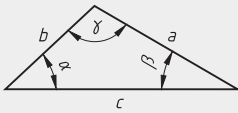
## Theorem of intersecting lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{d}{D}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

$$\frac{b_1}{d} = \frac{b_2}{D}$$

## Sum of angles in a triangle



- $a, b, c$  sides of the triangle  
 $\alpha, \beta, \gamma$  angles in the triangle

## Example:

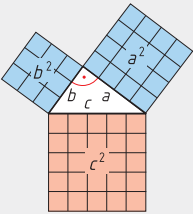
$$\begin{aligned} \alpha &= 21^\circ, \beta = 95^\circ, \gamma = ? \\ \gamma &= 180^\circ - \alpha - \beta = 180^\circ - 21^\circ - 95^\circ = 64^\circ \end{aligned}$$

## Sum of angles in a triangle

$$\alpha + \beta + \gamma = 180^\circ$$

In every triangle, the sum of the interior angles equals  $180^\circ$ .

## Pythagorean theorem



In a **right triangle** the square on the hypotenuse is equal to the sum of the squares on the sides meeting the right angle.

- $a$  side  
 $b$  side  
 $c$  hypotenuse

## 1st example:

$$\begin{aligned} c &= 35 \text{ mm}; a = 21 \text{ mm}; b = ? \\ b &= \sqrt{c^2 - a^2} = \sqrt{(35 \text{ mm})^2 - (21 \text{ mm})^2} = 28 \text{ mm} \end{aligned}$$

## 2nd example:

$$\begin{aligned} \text{CNC program where } R &= 50 \text{ mm and } I = 25 \text{ mm.} \\ K &= ? \\ c^2 &= a^2 + b^2 \\ R^2 &= I^2 + K^2 \\ K &= \sqrt{R^2 - I^2} = \sqrt{50^2 \text{ mm}^2 - 25^2 \text{ mm}^2} \\ K &= 43.3 \text{ mm} \end{aligned}$$

## Length of the hypotenuse

$$c^2 = a^2 + b^2$$

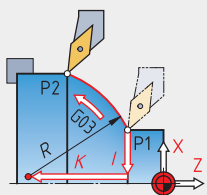
## Square on the hypotenuse

$$c = \sqrt{a^2 + b^2}$$

## Length of the sides

$$a = \sqrt{c^2 - b^2}$$

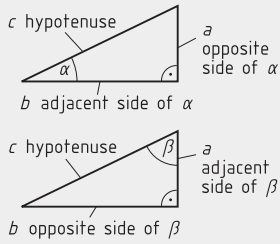
$$b = \sqrt{c^2 - a^2}$$



Functions of triangles

TM

Functions of right triangles (trigonometric functions)



$c$  hypotenuse (longest side)  
 $a, b$  sides:  
 -  $b$  is the adjacent side of  $\alpha$   
 -  $a$  is the opposite side of  $\alpha$   
 $\alpha, \beta, \gamma$  angles in the triangle,  $\gamma = 90^\circ$   
 $\sin$  notation of sine  
 $\cos$  notation of cosine  
 $\tan$  notation of tangent  
 $\sin \alpha$  sine of angle  $\alpha$

Trigonometric functions	
sine	= opposite side / hypotenuse
cosine	= adjacent side / hypotenuse
tangent	= opposite side / adjacent side
cotangent	= adjacent side / opposite side

1st example

$L_1 = 150 \text{ mm}, L_2 = 30 \text{ mm}, L_3 = 140 \text{ mm};$   
 angle  $\alpha = ?$

$$\tan \alpha = \frac{L_1 + L_2}{L_3} = \frac{180 \text{ mm}}{140 \text{ mm}} = 1.286$$

Angle  $\alpha = 52^\circ$

Relations applying to angle  $\alpha$ :

$\sin \alpha = \frac{a}{c}$	$\cos \alpha = \frac{b}{c}$	$\tan \alpha = \frac{a}{b}$
-----------------------------	-----------------------------	-----------------------------

Relations applying to angle  $\beta$ :

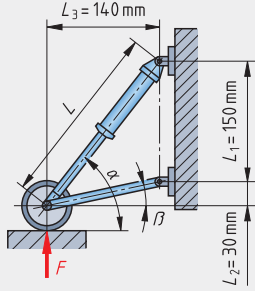
$\sin \beta = \frac{b}{c}$	$\cos \beta = \frac{a}{c}$	$\tan \beta = \frac{b}{a}$
----------------------------	----------------------------	----------------------------

2nd example

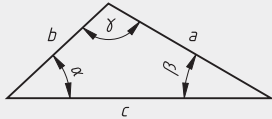
$L_1 = 150 \text{ mm}, L_2 = 30 \text{ mm}, \alpha = 52^\circ;$   
 Length of the shock absorber  $L = ?$

$$L = \frac{L_1 + L_2}{\sin \alpha} = \frac{180 \text{ mm}}{\sin 52^\circ} = 228.42 \text{ mm}$$

The calculation of an angle in degrees ( $^\circ$ ) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e. g. arc sine.



Functions of oblique triangles (law of sines, law of cosines)



According to the law of sines, the ratios of the sides correspond to the sine of their opposite angles in the triangle. If one side and two angles are known, the other values can be calculated with the help of this function.

Law of sines
$a : b : c = \sin \alpha : \sin \beta : \sin \gamma$
$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$

Side  $a \rightarrow$  opposite angle  $\alpha$   
 Side  $b \rightarrow$  opposite angle  $\beta$   
 Side  $c \rightarrow$  opposite angle  $\gamma$

There are many transformation options:

Example

$F = 800 \text{ N}, \alpha = 40^\circ, \beta = 38^\circ; F_z = ?, F_d = ?$

The forces are calculated with the help of the forces diagram.

$$\frac{F}{\sin \alpha} = \frac{F_z}{\sin \beta} \Rightarrow F_z = \frac{F \cdot \sin \beta}{\sin \alpha}$$

$$F_z = \frac{800 \text{ N} \cdot \sin 38^\circ}{\sin 40^\circ} = 766.24 \text{ N}$$

$$\frac{F}{\sin \alpha} = \frac{F_d}{\sin \varphi} \Rightarrow F_d = \frac{F \cdot \sin \varphi}{\sin \alpha}$$

$$F_d = \frac{800 \text{ N} \cdot \sin 102^\circ}{\sin 40^\circ} = 1217.38 \text{ N}$$

$a = \frac{b \cdot \sin \alpha}{\sin \beta} = \frac{c \cdot \sin \alpha}{\sin \gamma}$
$b = \frac{a \cdot \sin \beta}{\sin \alpha} = \frac{c \cdot \sin \beta}{\sin \gamma}$
$c = \frac{a \cdot \sin \gamma}{\sin \alpha} = \frac{b \cdot \sin \gamma}{\sin \beta}$

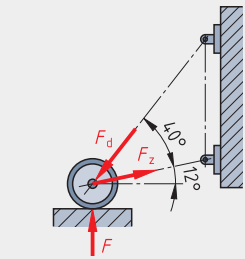
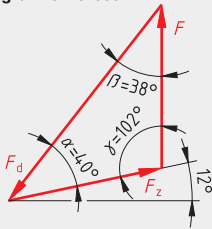


Diagram of forces



Law of cosines
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$

The calculation of an angle in degrees ( $^\circ$ ) or as a circular measure (rad) is done with the help of inverse trigonometric functions, e.g. arc cos.

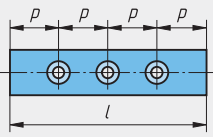
Transformation, e. g.

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$$

## Division of lengths, arc length, composite length

### Sub-dividing length

#### Edge distance = spacing



$l$  total length       $n$  number of holes  
 $p$  division

#### Example:

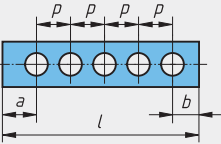
$$l = 2 \text{ m}; n = 24 \text{ holes}; p = ?$$

$$p = \frac{l}{n+1} = \frac{2000 \text{ mm}}{24+1} = 80 \text{ mm}$$

#### Pitch

$$p = \frac{l}{n+1}$$

#### Edge distance $\neq$ spacing



$l$  total length       $n$  number of holes  
 $p$  spacing       $a, b$  edge distances

#### Example:

$$l = 1950 \text{ mm}; a = 100 \text{ mm}; b = 50 \text{ mm};$$

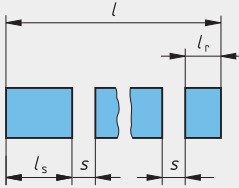
$$n = 25 \text{ holes}; p = ?$$

$$p = \frac{l - (a + b)}{n - 1} = \frac{1950 \text{ mm} - 150 \text{ mm}}{25 - 1} = 75 \text{ mm}$$

#### Pitch

$$p = \frac{l - (a + b)}{n - 1}$$

#### Subdividing into pieces



$l$  bar length       $s$  saw cutting width  
 $z$  number of pieces       $l_r$  remaining length  
 $l_s$  piece length

#### Example:

$$l = 6 \text{ m}; l_s = 230 \text{ mm}; s = 1.2 \text{ mm}; z = ?; l_r = ?$$

$$z = \frac{l}{l_s + s} = \frac{6000 \text{ mm}}{230 \text{ mm} + 1.2 \text{ mm}} = 25.95 = 25 \text{ pieces}$$

$$l_r = l - z \cdot (l_s + s) = 6000 \text{ mm} - 25 \cdot (230 \text{ mm} + 1.2 \text{ mm}) = 220 \text{ mm}$$

#### Number of pieces

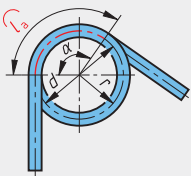
$$z = \frac{l}{l_s + s}$$

#### Remaining length

$$l_r = l - z \cdot (l_s + s)$$

### Arc length

#### Example: Torsion spring



$l_a$  arc length       $\alpha$  angle at centre  
 $r$  radius       $d$  diameter

#### Example:

$$r = 36 \text{ mm}; \alpha = 120^\circ; l_a = ?$$

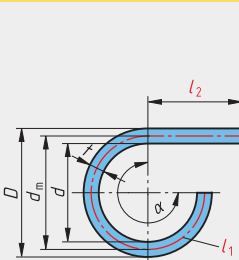
$$l_a = \frac{\pi \cdot r \cdot \alpha}{180^\circ} = \frac{\pi \cdot 36 \text{ mm} \cdot 120^\circ}{180^\circ} = 75.36 \text{ mm}$$

#### Arc length

$$l_a = \frac{\pi \cdot r \cdot \alpha}{180^\circ}$$

$$l_a = \frac{\pi \cdot d \cdot \alpha}{360^\circ}$$

### Composite length



$D$  outside diameter       $d$  inside diameter  
 $d_m$  mean diameter       $t$  thickness  
 $l_1, l_2$  section lengths       $L$  composite length  
 $\alpha$  angle at centre      length

#### Example (composite length, picture left):

$$D = 360 \text{ mm}; t = 5 \text{ mm}; \alpha = 270^\circ; l_2 = 70 \text{ mm};$$

$$d_m = ?; L = ?$$

$$d_m = D - t = 360 \text{ mm} - 5 \text{ mm} = 355 \text{ mm}$$

$$L = l_1 + l_2 = \frac{\pi \cdot d_m \cdot \alpha}{360^\circ} + l_2$$

$$= \frac{\pi \cdot 355 \text{ mm} \cdot 270^\circ}{360^\circ} + 70 \text{ mm} = 906.45 \text{ mm}$$

#### Composite length

$$L = l_1 + l_2 + \dots$$